

(1) 試判斷下列各組數是否互質，互質請打✓

- (1) (✓) 32 和 51
 2^5 3×17
 (2) () 143 和 195
 11×13 $3 \times 5 \times 13$
 (3) (✓) 85 和 58
 5×17 2×29
 (4) () 84 和 91
 $2 \times 3 \times 7$ 7×13
 (5) () 57 和 75
 3×19 3×5^2
 (6) () 102 和 221
 $2 \times 3 \times 17$ 13×17

(2) 下列哪些數與 78 互質？ 221、175、285、539

$$78 = 2 \times 3 \times 13$$

$$221 = 13 \times 17 \Rightarrow (78, 221) = 13 \quad \times$$

$$175 = 5^2 \times 7 \Rightarrow (78, 175) = 1 \quad \checkmark$$

$$285 = 3 \times 5 \times 19 \Rightarrow (78, 285) = 3 \quad \times$$

$$539 = 7 \times 11 \Rightarrow (78, 539) = 1 \quad \checkmark \quad A: 175, 539$$

(3) 請寫出比 30 小，且與 42 互質的所有正整數

$$42 = 2 \times 3 \times 7$$

(去掉 2 倍, 3 倍, 7 倍)

$$\Rightarrow 1, 5, 11, 13, 17, 19, 23, 25, 29$$

求(4)~(18)題中各組數最大公因數、最小公倍數

(答案不需乘出來，用標準分解式表示即可)

(4) 64, 184

(5) 57, 133

$$\begin{array}{l} 2 \overline{) 64, 184} \\ 2 \overline{) 32, 92} \\ 2 \overline{) 16, 46} \\ 8, 23 \end{array} \quad (64, 184) = 2^3$$

$$[64, 184] = 2^6 \times 23$$

$$\begin{array}{l} 19 \overline{) 57, 133} \\ 3, 7 \end{array} \quad (57, 133) = 19$$

$$[57, 133] = 3 \times 7 \times 19$$

(6) 7, 23

(7) 12, 15, 18

$$\begin{array}{l} 1 \overline{) 7, 23} \\ 7, 23 \end{array}$$

$$(7, 23) = 1$$

$$[7, 23] = 7 \times 23$$

$$\begin{array}{l} \text{三者} \rightarrow 3 \overline{) 12, 15, 18} \\ \text{二者} \rightarrow 2 \overline{) 4, 5, 6} \\ 2, 5, 3 \end{array}$$

$$(12, 15, 18) = 3$$

$$[12, 15, 18] = 2^2 \times 3^2 \times 5$$

(8) 95, 190, 490

(9) $a = 3^2 \times 5^3$, $b = 3^3 \times 7$

$$\begin{array}{l} \text{三者} \rightarrow 5 \overline{) 95, 190, 490} \\ \text{二者} \rightarrow 19 \overline{) 19, 38, 98} \\ \text{二者} \rightarrow 2 \overline{) 1, 2, 98} \\ 1, 1, 49 \end{array}$$

$$(95, 190, 490) = 5$$

$$[95, 190, 490] = 2 \times 5 \times 7^2 \times 19$$

$$(a, b) = 3^2$$

$$[a, b] = 3^3 \times 5^3 \times 7$$

(10) $a = 2^2 \times 3^3 \times 5 \times 7$, $b = 2^3 \times 3^2 \times 7 \times 11$

$$(a, b) = 2^2 \times 3^2 \times 7$$

$$[a, b] = 2^3 \times 3^3 \times 5 \times 7 \times 11$$

(11) $a = 2^4 \times 3^2 \times 5^2$, $b = 2^5 \times 3^3 \times 5$, $c = 2^2 \times 3^3 \times 5 \times 7$

$$(a, b, c) = 2^2 \times 3^2 \times 5$$

$$[a, b, c] = 2^5 \times 3^3 \times 5^2 \times 7$$

(12) $a = 3^3 \times 13 \times 23$, $b = 3^4 \times 7 \times 13^2 \times 23$, $c = 3^2 \times 5 \times 7^3 \times 13$

$$(a, b, c) = 3^2 \times 13$$

$$[a, b, c] = 3^4 \times 5 \times 7^3 \times 13^2 \times 23$$

(13) $a = 2^3 \times 17^2$, $b = 7^3 \times 23$

$$(a, b) = 1$$

$$[a, b] = 2^3 \times 7^3 \times 17^2 \times 23$$

(14) $a = 2^2 \times 3 \times 5^2 \times 11$, $b = 555$

$$b = 3 \times 5 \times 37$$

$$(a, b) = 3 \times 5$$

$$[a, b] = 2^2 \times 3 \times 5^2 \times 11 \times 37$$

(15) $a = 4 \times 9 \times 11$, $b = 10 \times 15 \times 19$

$$a = 2^2 \times 3^2 \times 11$$

$$b = 2 \times 5 \times 3 \times 5 \times 19$$

$$= 2 \times 3 \times 5^2 \times 19$$

$$(a, b) = 2 \times 3$$

$$[a, b] = 2^2 \times 3^2 \times 5^2 \times 11 \times 19$$

(16) $a = 1 \times 2 \times 3 \times 4 \times 5 \times 6$, $b = 7 \times 8 \times 9 \times 10 \times 11 \times 12$

$$a = 2^4 \times 3^2 \times 5$$

$$b = 2^6 \times 3^3 \times 5 \times 7 \times 11$$

$$(a, b) = 2^4 \times 3^2 \times 5$$

$$[a, b] = 2^6 \times 3^3 \times 5 \times 7 \times 11$$

(17) 甲 = $13 \times 15 \times 21 \times 27$, 乙 = $9 \times 11 \times 18 \times 24$
 $3 \times 5 \quad 3 \times 7 \quad 3^3 \quad 3^2 \quad 2 \times 3^2 \quad 2^3 \times 3$

甲 = $3^5 \times 5 \times 7 \times 13$

乙 = $2^4 \times 3^5 \times 11$

$(甲, 乙) = 3^5$

$[甲, 乙] = 2^4 \times 3^5 \times 5 \times 7 \times 11 \times 13$

(18) $a = 195$, $b = 3^2 \times 5^3 \times 14$, $c = 2^2 \times 3^3 \times 65$
 $2 \times 7 \quad 5 \times 13$

$a = 3 \times 5 \times 13$

$b = 2 \times 3^2 \times 5^3 \times 7$ $(a, b, c) = 3 \times 5$

$c = 2^2 \times 3^3 \times 5 \times 13$ $[a, b, c] = 2^2 \times 3^3 \times 5^3 \times 7 \times 13$

(19) 若 $a = 14$, $b = 15$, 則 $(a, b) \times [a, b] =$ _____

$(14, 15) = 1$

$[14, 15] = 2 \times 3 \times 5 \times 7$
 $= 210$

$\Rightarrow 1 \times 210 = 210$

(20) 已知兩正整數 a, b , 其中 $a \times b = 195$, 且 $(a, b) = 3$,
 求 $[a, b] =$ _____

$\therefore (a, b) \times [a, b] = a \times b$

$\therefore 3 \times [a, b] = 195$

$[a, b] = 65$

(21) a 是一個正整數, 其所有正因數有 $1, 2, 4, 7, 14, 28$,
 則 a 與 70 的最大公因數為何?

$a = 1 \times 28 = 28$

$(28, 70) = 2 \times 7$

$= 14$

$2 \mid 28, 70$
 $7 \mid 14, 35$
 $2, 5$

(22) 147 和 189 的最大公因數 = 21

147 和 189 的所有公因數有 1, 3, 7, 21

$3 \mid 147, 189$
 $7 \mid 49, 63$
 $7, 9$

$(147, 189)$

$= 3 \times 7 = 21$

* 所有公因數為最大公因數
 的所有因數

$\Rightarrow 1, 3, 7, 21$

(23) 54 和 72 的最大公因數和最小公倍數的和為多少?

$2 \mid 54, 72$
 $3 \mid 27, 36$
 $3 \mid 9, 12$
 $3, 4$

$(54, 72) = 2 \times 3^2 = 18$

$[54, 72] = 2^3 \times 3^3 = 216$

$\Rightarrow 18 + 216 = 234$

(24) 甲數 = $(63, 231)$, 則甲數所有因數的和為多少?

甲 = $(63, 231)$

$= 3 \times 7$

$= 21$

$\Rightarrow 21$ 因: $1, 3, 7, 21$

$\Rightarrow 1 + 3 + 7 + 21 = 32$

$3 \mid 63, 231$
 $7 \mid 21, 33$
 $3, 11$

(25) 如右圖,

1. $(a, b) = ?$

2. $[a, b] = ?$

3. $b = ?$

$5 \mid \begin{matrix} 140 & 105 \\ a & b \end{matrix}$
 $7 \mid \begin{matrix} 28 & c \end{matrix}$
 $d \quad 3$
 4

1. $(a, b) = 5 \times 7 = 35$

2. $d = \frac{28}{7} = 4$

3. $c = 7 \times 3 = 21$

$b = 21 \times 5 = 105$

$[a, b] = 2^3 \times 3 \times 5 \times 7$
 $= 420$

(26) 設 $A = 2^9 \times 3^{12} \times 7^4$, $B = 2^8 \times 3^6 \times 11^4$,

若 $(A, B) = 2^a \times 3^b$, $[A, B] = 2^m \times 3^n \times 7^p \times 11^q$,

求: (1) $a + b = ?$ (2) $m \times n - p \div q = ?$

(1) $(A, B) = 2^8 \times 3^6$

$\Rightarrow a = 8, b = 6$

$\Rightarrow 8 + 6 = 14$

(2) $[A, B] = 2^9 \times 3^{12} \times 7^4 \times 11^4$

$\Rightarrow m = 9, n = 12, p = 4, q = 4$

$\Rightarrow 9 \times 12 - 4 \div 4$

$= 108 - 1$

$= 107$

(27) 若 360 和 378 的最大公因數為 $2^a \times b^2$, 最小公倍數為

$2^3 \times 3^c \times d \times e$ (都用標準分解式表示), 則 a, b, c, d, e

分別為何?

$(360, 378) = 2 \times 3^2$

$\Rightarrow a = 1, b = 3$

$[360, 378] = 2^3 \times 3^3 \times 5 \times 7$

$\Rightarrow c = 3, d = 5, e = 7$

$2 \mid 360, 378$
 $3 \mid 180, 189$
 $3 \mid 60, 63$
 $20, 21$
 $\downarrow \quad \downarrow$
 $2^2 \times 5 \quad 3 \times 7$

$A: a = 1, b = 3, c = 3, d = 5, e = 7$