

答案

一、基礎題：

1. C 2. D 3. D 4. D 5. A 6. C 7. C 8. D
9. A 10. B 11. C 12. A 13. D 14. A 15. B 16. C
17. B 18. C 19. A 20. B

二、精熟題：

21. B 22. B 23. B

三、非選擇題：

1. (1) 0 或 -6
(2) 當 $m=0$ 時，另一根為 -1；當 $m=-6$ 時，另一根為 17
2. 100 元

詳解

一、基礎題：

$$1. \left(\frac{x}{3} - 1\right)(x - \frac{1}{3}) = 0 \Rightarrow \frac{x}{3} - 1 = 0 \text{ 或 } x - \frac{1}{3} = 0$$

$$\Rightarrow x = 3 \text{ 或 } x = \frac{1}{3} \quad \therefore a = 3, b = \frac{1}{3}$$

$$\text{故 } 2a + b = 6 + \frac{1}{3} = 6\frac{1}{3}$$

$$2. (A) 2(x+2)(x-3) = 2x^2 - 2x - 12 \text{ 為一元二次多項式}$$

$$(B) (x+2)(x-3) = (x+2)(x+3)$$

$$\Rightarrow x^2 - x - 6 = x^2 + 5x + 6 \text{ 為一元一次方程式}$$

$$(C) 2(x+2)(x-3) = 2x^2 + 5x - 6$$

$$\Rightarrow 2x^2 - 2x - 12 = 2x^2 + 5x - 6 \text{ 為一元一次方程式}$$

$$(D) (x+2)(x-3) = 2(x+2)(x-3)$$

$$\Rightarrow x^2 - x - 6 = 2x^2 - 2x - 12$$

$$\Rightarrow x^2 - x - 6 = 0 \text{ 為一元二次方程式}$$

$$3. x = 5 \text{ 代入 } \Rightarrow 2 \times 5^2 + k \times 5 + 5 = 0$$

$$5k = -55, k = -11$$

$$4. x = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-4)}}{2 \times 2} = \frac{-3 \pm \sqrt{41}}{4}$$

$$\text{令 } a = \frac{-3 + \sqrt{41}}{4}, b = \frac{-3 - \sqrt{41}}{4}$$

$$\text{故 } |a| + |b| = \left| \frac{-3 + \sqrt{41}}{4} \right| + \left| \frac{-3 - \sqrt{41}}{4} \right| \\ = \frac{-3 + \sqrt{41}}{4} + \frac{3 + \sqrt{41}}{4} = \frac{\sqrt{41}}{2}$$

$$5. (x+4)(3x-1) = -6x^2 + 35x - 20$$

$$3x^2 + 11x - 4 = -6x^2 + 35x - 20$$

$$9x^2 - 24x + 16 = 0$$

$$(3x-4)^2 = 0 \Rightarrow x = \frac{4}{3} \text{ (重根)}$$

$$6. x^2 - 2x + 1 = 2024 + 1, (x-1)^2 = 2025$$

$$x-1 = \pm 45, x = 1 \pm 45, x = -44 \text{ 或 } x = 46$$

$$7. (x-5)(x+2) = 0, x^2 - 3x - 10 = 0$$

$$\text{比較係數可得 } a = 1, b = -3$$

$$\therefore a + b = 1 + (-3) = -2$$

$$8. (4x+6)(x+1) = (2x+3)(x+1)$$

$$\Rightarrow 2(2x+3)(x+1) = (2x+3)(x+1)$$

$$\Rightarrow (2x+3)(x+1) = 0$$

$$\Rightarrow 2x+3 = 0 \text{ 或 } x+1 = 0$$

$$\Rightarrow x = -\frac{3}{2} \text{ 或 } x = -1, \text{ 皆是負根}$$

$$9. k = \frac{-7 - \sqrt{7^2 - 4 \times 1 \times (-7)}}{2} \\ = \frac{-7 - \sqrt{77}}{2} \div \frac{-7 - 8.775}{2} \div -7.8875$$

$$10. 2x(x-4) - a + 7 = 0 \Rightarrow 2x^2 - 8x + (-a+7) = 0$$

$\therefore$ 方程式有解

$$\therefore \text{判別式} = (-8)^2 - 4 \times 2 \times (-a+7) \geq 0$$

$$64 + 8a - 56 \geq 0 \Rightarrow a \geq -1$$

$$11. 10x - 25 = x^2 - 3^2, x^2 - 10x + 16 = 0$$

$$(x-2)(x-8) = 0, x = 2 \text{ 或 } x = 8$$

$$\text{故和} = 2 + 8 = 10$$

$$12. \text{判別式} = 4^2 - 4 \times k \times \frac{5k-3}{2} = 0, 16 - 10k^2 + 6k = 0$$

$$5k^2 - 3k - 8 = 0, (5k-8)(k+1) = 0$$

$$\therefore k = \frac{8}{5} \text{ 或 } k = -1$$

$$\text{故 } |a-b| = \left| \frac{8}{5} - (-1) \right| = \frac{13}{5} = 2.6$$

$$13. (x-29)^2 = k \Rightarrow x-29 = \pm \sqrt{k} \Rightarrow x = 29 \pm \sqrt{k}$$

$$\text{又 } 1 < k < 4 \Rightarrow 1 < \sqrt{k} < 2$$

$$\therefore a = 29 + \sqrt{k} \quad \therefore 30 < a < 31$$

$$\therefore b = 29 - \sqrt{k} \quad \therefore 27 < b < 28, \text{ 故選(D)}$$

$$14. \text{原式} \times 10 \text{ 得 } 3x^2 - 20x - 32 = 0$$

$$(3x+4)(x-8) = 0, x = -\frac{4}{3} \text{ 或 } x = 8$$

$$\text{故 } |a-b| = \left| 8 - \left(-\frac{4}{3}\right) \right| = \frac{28}{3}$$

$$15. 3x^2 - 8x + k = 0 \Rightarrow x^2 - \frac{8}{3}x = -\frac{k}{3}$$

$$x^2 - \frac{8}{3}x + \left(\frac{4}{3}\right)^2 = -\frac{k}{3} + \left(\frac{4}{3}\right)^2$$

$$\left(x - \frac{4}{3}\right)^2 = \frac{16-3k}{9}, x - \frac{4}{3} = \pm \frac{\sqrt{16-3k}}{3}$$

$$x = \frac{4 \pm \sqrt{16-3k}}{3} \Rightarrow 16-3k = 13 \Rightarrow k = 1$$

$$16. x^2 = 11881, x = \pm \sqrt{11881} \quad \therefore \sqrt{11881} = 109$$

$$x^2 + 6x - 11872 = 0 \Rightarrow x^2 + 6x + 9 = 11872 + 9$$

$$x^2 + 6x + 9 = 11881 \Rightarrow x+3 = \pm \sqrt{11881} \Rightarrow x = -3 \pm 109$$

$$\therefore x = 106 \text{ 或 } x = -112$$

17. 由題意知：

$$(x^2 - 12x - 40) + (3x + 4) = 0$$

$$\therefore x^2 - 9x - 36 = 0$$

$$(x+3)(x-12) = 0 \Rightarrow x = -3 \text{ 或 } x = 12$$

$$\text{故所求} = (-3) + 12 = 9$$

$$18. x^2 - 5x = 18 - x^2, 2x^2 - 5x - 18 = 0$$

$$(2x-9)(x+2) = 0$$

$$x = \frac{9}{2} \text{ (不合) 或 } x = -2$$

$$19. \text{設三數分別為 } x-2, x, x+2$$

$$(x-2)^2 + x^2 + (x+2)^2 = 683$$

$$3x^2 + 8 = 683 \Rightarrow x^2 = 225 \Rightarrow x = \pm 15 \text{ (負不合)}$$

$$\therefore \text{最大的奇數} = 15 + 2 = 17$$

$$20. \text{小翰: } (x-5)(x-4) = 0 \Rightarrow x^2 - 9x + 20 = 0$$

$$\text{小林: } (x-4)(x-2) = 0 \Rightarrow x^2 - 6x + 8 = 0$$

$$\therefore \text{正確的方程式為 } x^2 - 9x + 8 = 0$$

二、精熟題：

21. 設 $\overline{FC} = \overline{EC} = x$

$$\frac{(x+3)(x+3)}{2} = x^2$$

$$x^2 - 6x - 9 = 0$$

$$x = 3 + 3\sqrt{2} \text{ 或 } x = 3 - 3\sqrt{2} \text{ (負不合)}$$

22. $(a+2) \times 3a = 3a^2 + 6a$

$$\frac{3a \times 2}{2} + \frac{(2a+2) \times a}{2} +$$

$$\frac{(a-2)(a+2)}{2} = \frac{3}{2}a^2 + 4a - 2$$

$$(3a^2 + 6a) - \left(\frac{3}{2}a^2 + 4a - 2\right) = 18$$

$$3a^2 + 4a - 32 = 0$$

$$(3a-8)(a+4) = 0$$

$$a = \frac{8}{3} \text{ 或 } a = -4 \text{ (不合)}$$

23. 設路寬為 x 公尺

$$\text{則 } (40-3x)(30-3x) + 384 = 40 \times 30$$

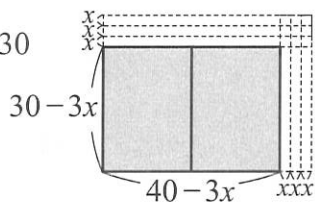
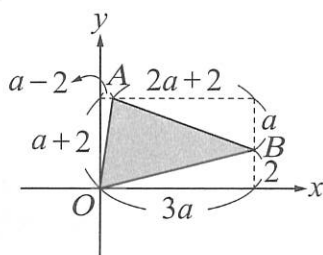
$$\Rightarrow 9x^2 - 210x + 384 = 0$$

$$\Rightarrow 3x^2 - 70x + 128 = 0$$

$$\Rightarrow (x-2)(3x-64) = 0$$

$$\Rightarrow x = 2 \text{ 或 } x = \frac{64}{3} \text{ (不合)}$$

故路寬為 2 公尺



三、非選擇題：

1. (1) 將 $x=2$ 代入得

$$2^2 + (3m-1) \times 2 + (m^2-2) = 0$$

$$m^2 + 6m = 0, m(m+6) = 0$$

$$m = 0 \text{ 或 } m = -6$$

(2) ① 當 $m=0$ 時，方程式為

$$x^2 + (3 \times 0 - 1)x + (0^2 - 2) = 0$$

$$x^2 - x - 2 = 0$$

$$(x+1)(x-2) = 0$$

$$x = -1 \text{ 或 } x = 2 \Rightarrow \text{另一根為 } -1$$

② 當 $m=-6$ 時，方程式為

$$x^2 + [3 \times (-6) - 1]x + [(-6)^2 - 2] = 0$$

$$x^2 - 19x + 34 = 0$$

$$(x-2)(x-17) = 0$$

$$x = 2 \text{ 或 } x = 17 \Rightarrow \text{另一根為 } 17$$

答：(1) 0 或 -6；

(2) 當 $m=0$ 時，另一根為 -1；當 $m=-6$ 時，另一根為 17

2. 設降價 $20x$ 元，多賣 $10x$ 個， x 為整數

$$(350 - 20x)(100 + 10x) = 37500$$

$$2x^2 - 15x + 25 = 0$$

$$(2x-5)(x-5) = 0$$

$$x = \frac{5}{2} \text{ (不合) 或 } x = 5$$

$$\therefore \text{每個降價 } 20 \times 5 = 100 \text{ (元)}$$

答：100 元