

1. 如圖, $\overline{AD}$ 、 $\overline{AE}$ 、 $\overline{BC}$ 分別與圓切於 D 、 E 、 F 三點, 若切線段 $\overline{AE}$ 為 8, 則 $\triangle ABC$ 的周長=?

$$\because \overline{BF} = \overline{BD} \\ \overline{CF} = \overline{CE} \quad (\text{切線長相等})$$

$$\text{且 } \overline{AD} = \overline{AE} = 8$$

$$\therefore \triangle ABC \text{ 周長} = \overline{AB} + \overline{BC} + \overline{AC} \\ = \overline{AB} + \overline{BF} + \overline{CF} + \overline{AC} \\ = \overline{AB} + \overline{BD} + \overline{CE} + \overline{AC} \\ = \overline{AD} + \overline{AE} = 16$$

2. 坐標平面上有一圓, 圓心為 $P(2, 2)$, 半徑為 $2\sqrt{2}$, 若直線 L 的方程式為 $x = -1$, 則 L 與圓 P 有幾個交點?

P 到 L 的距離

$$\because 3 > 2\sqrt{2}$$

$$= 2 - (-1)$$

$$= 3$$

$\therefore L$ 與圓 P 沒有交點

3. 如圖, 圓外切四邊形 $ABCD$ 中, 已知 $\overline{AB} = 12$, $\overline{BC} = 5 + x$, $\overline{CD} = 3x + 1$, $\overline{AD} = x^2$, (1) 求 x 值。(2) 四邊形 $ABCD$ 周長。

(1) $\because$ 圓外切四邊形

$$\therefore x^2 + 5 + x = 12 + 3x + 1$$

$$x^2 - 2x - 8 = 0$$

$$(x - 4)(x + 2) = 0$$

$$x = 4 \text{ or } -2 (\text{捨})$$

$$\Rightarrow x = 4$$

$$(2) \overline{AD} = 16, \overline{DC} = 13, \overline{BC} = 9 \\ \Rightarrow 16 + 13 + 9 + 12 = 50$$

4. 如圖, 菱形 $ABCD$ 的周長 60cm , $\overline{AC} = 24\text{cm}$, 圓 O 為其內切圓, 求圓 O 的直徑

$$\overline{AB} = \frac{60}{4} = 15$$

$$\overline{OE} = \frac{\frac{1}{2} \times 12}{1 + 5} = \frac{36}{5}$$

$$\overline{AO} = \frac{24}{2} = 12$$

$$= \frac{36}{5}$$

$$\Rightarrow \overline{BO} = \sqrt{15^2 - 12^2} \Rightarrow \text{直徑} = \frac{36}{5} \times 2 = \frac{72}{5}$$

$$A: \frac{72}{5} \text{ cm}$$

5. 如圖, 圓外切四邊形 $ABCD$ 的周長 100cm ,

若 $\overline{AB} : \overline{BC} : \overline{CD} = 6 : 3 : 4$, 則 $\overline{AD} = ?$

$$\text{設 } \overline{AB} = 6x, \overline{BC} = 3x, \overline{CD} = 4x$$

$$\therefore \overline{AD} + 3x = 6x + 4x$$

$$\therefore \overline{AD} = 7x$$

$$\Rightarrow 7x + 4x + 3x + 6x = 100$$

$$20x = 100 \rightarrow \overline{AD} = 7 \times 5$$

$$x = 5 \rightarrow \overline{AD} = 35(\text{cm})$$

6. 如圖, 兩圓外切於 P , $\overline{AD}$ 、 $\overline{AB}$ 、 $\overline{BC}$ 、 $\overline{CD}$ 、 $\overline{EF}$ 均為兩圓的切線, 若 $\overline{AD} = 12$, $\overline{BC} = 11$, $\overline{AB} = 8$, $\overline{CD} = 5$, 則 $\overline{EF} = ?$

$$\therefore \overline{AD} + \overline{BC} = \overline{AE} + \overline{DE} + \overline{BF} + \overline{CF} \\ = \overline{AB} + \overline{EF} + \overline{EF} + \overline{DC} \\ = \overline{AB} + \overline{CD} + 2\overline{EF}$$

$$\therefore 12 + 11 = 8 + 5 + 2\overline{EF}$$

$$\overline{EF} = 5$$

7. 如圖, 已知梯形 $ABCD$ 外切於圓 O , 且 $\overline{AB} = \overline{CD}$, $\overline{AD} = 4$, $\overline{BC} = 6$, 求 (1) 梯形 $ABCD$ 面積 (2) 圓 O 面積

$$(1) \overline{AB} = \overline{CD} = \frac{4+6}{2} = 5$$

$$\overline{HB} = \overline{KC} = \frac{6-4}{2} = 1$$

$$\overline{AH} = \sqrt{5^2 - 1^2} = 2\sqrt{6}$$

$$= \sqrt{24} = 2\sqrt{6}$$

$$(2) \text{半徑} = \frac{2\sqrt{6}}{2} = \sqrt{6}$$

$$(\sqrt{6})^2 \times \pi = 6\pi$$

8. 如圖, 四邊形 $ABCD$ 為矩形, 且梯形 $ABED$ 的四邊分別與圓 O 相切, 若 $\overline{CD} = 12$, $\overline{DE} = 15$,

求 (1) $\overline{BE}$ (2) 灰色區域的面積

$$(1) \overline{CE} = \sqrt{15^2 - 12^2} = 9$$

$$\Rightarrow \overline{BE} = 9$$

$$\text{設 } \overline{BE} = x$$

$$\overline{AD} = x + 9$$

$$\Rightarrow x + 9 + x = 12 + 15$$

$$2x = 18$$

(2) 灰色區域

$$= \square ABCD - \triangle CDE - \text{圓 } O$$

$$= 12 \times 18 - \frac{1}{2} \times 9 \times 12 - 6^2 \times \pi = 162 - 36\pi$$

9. 已知 $\overline{AB}$ 的長度為 10, $\overline{CD}$ 的長度為 8, 圓 O_2 的半徑為 12, 且 $\angle AO_1B = \angle CO_2D$, 求圓 O_1 的半徑。

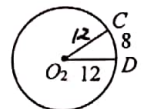
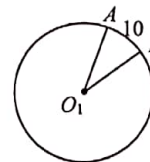
$$\therefore \angle AO_1B = \angle CO_2D$$

$$\therefore \overline{AB} : \overline{CD} = \overline{AO_1} : \overline{CO_2}$$

$$\Rightarrow \frac{10}{8} = \frac{\overline{AO_1}}{12}$$

$$4\overline{AO_1} = 60$$

$$\overline{AO_1} = 15$$



圓 O_1 半徑 = 15

10. 如圖, 圓上兩點 A 、 B 把圓 O 分成大小兩弧, 大弧的度數比小弧度數的 3 倍多 20° , 則 $\angle AOB = ?$

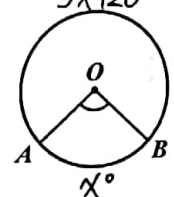
$$\text{設小弧 } x^\circ, \text{ 大弧 } (3x + 20)^\circ$$

$$x + 3x + 20 = 360$$

$$4x = 340$$

$$x = 85$$

$$\Rightarrow \angle AOB = 85^\circ$$



11. 如圖， $\angle AOB = 120^\circ$ ，圓 O 半徑 8 公分，則

(1) $\widehat{ACB}$ 的度數是多少？ (2) $\widehat{ADB}$ 的度數是多少？

(3) $\widehat{AB} = ?$ (4) $\widehat{ACB}$ 的長度 (5) 扇形 AOB 周長

(6) 扇形 AOB 面積

$$(1) \widehat{ACB} = \angle AOB = 120^\circ$$

$$(2) \widehat{ADB} = 360^\circ - 120^\circ = 240^\circ$$

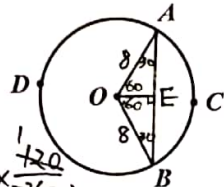
$$(3) \widehat{AE} = \frac{8}{2} \times \sqrt{3} = 4\sqrt{3}$$

$$\Rightarrow \widehat{AB} = 4\sqrt{3} \times 2 = 8\sqrt{3} \text{ (cm)}$$

$$(4) 8 \times 2 \times \pi \times \frac{120}{360} = \frac{16}{3} \pi \text{ (cm)}$$

$$(5) \frac{16}{3} \pi + 8 + 8 = \frac{16}{3} \pi + 16 \text{ (cm)}$$

$$(6) 8 \times 8 \times \pi \times \frac{1}{3} = \frac{64}{3} \pi \text{ (cm}^2\text{)}$$



12. 如圖兩同心圓，已知半徑分別為 4 cm 及 7 cm，已知

$\widehat{CD} = 60^\circ$ ，求 (1) $\angle AOB = ?$ (2) 斜線部分面積

(3) 斜線部分周長

$$(1) \because \angle COD = \widehat{CD} = 60^\circ$$

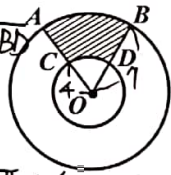
$$\therefore \angle AOB = 60^\circ$$

$$(3) \widehat{AB} + \widehat{CD} + \widehat{AC} + \widehat{BD} = 7 \times \frac{1}{2} \pi \times \frac{1}{3} + 4 \times \frac{1}{2} \pi \times \frac{1}{3} + 2(7-4) = \frac{11}{3} \pi + 6 \text{ (cm)}$$

$$(2) \Delta AOB - \Delta COD$$

$$= 7^2 \pi \times \frac{1}{6} - 4^2 \pi \times \frac{1}{6}$$

$$= \frac{49}{6} \pi - \frac{16}{6} \pi = \frac{33}{6} \pi = \frac{11}{2} \pi \text{ (cm}^2\text{)}$$



13. 如圖兩同心圓，已知 $\widehat{CD} = 4\pi$ cm，且 $\widehat{AC} = \widehat{CO}$ ， $\widehat{AO} = 18$

求 (1) $\angle AOB = ?$ (2) $\widehat{AB}$ 長度？

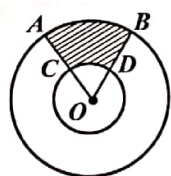
$$(1) \text{ 設 } \angle AOB = x^\circ$$

$$\widehat{CD} = \frac{18}{2} = 9$$

$$\widehat{CD} = 4 \times \frac{1}{2} \pi \times \frac{x}{360} = 4\pi$$

$$\frac{x}{40} = 2$$

$$x = 80 \Rightarrow \angle AOB = 80^\circ$$



$$(2) \widehat{AB} = 7 \times 2 \times \pi \times \frac{80}{360} = 8\pi \text{ (cm)}$$

14. 如圖， P 為圓 O 外一點，割線 PB 通過圓心 O ，且交圓 O

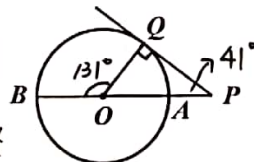
於 A 及 B ，又 PQ 為切線， Q 為切點。若 $\angle QPB = 41^\circ$ ，

則 $\widehat{BQ}$ 的度數是多少

連 QO

$$\Rightarrow \widehat{BQ} = \angle QOB$$

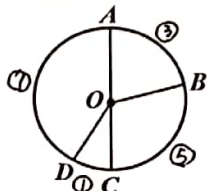
$$\angle QOB = 90^\circ + 41^\circ = 131^\circ$$



15. 如圖， $\widehat{AB} : \widehat{BC} : \widehat{CD} : \widehat{DA} = 3 : 5 : 1 : 7$ ，圓 O 的半徑為 8 公分，則 (1) $\angle COD = ?$ (2) $\widehat{BC}$ 的長度

$$(1) \angle COD = 360^\circ \times \frac{1}{16} = 22.5^\circ$$

$$(2) 8 \times 2 \times \pi \times \frac{5}{16} = 5\pi \text{ (cm)}$$



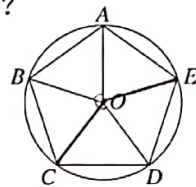
16. 如圖，正五邊形 $ABCDE$ 的頂點皆在圓 O 上，且圓半徑為 10，則：

(1) $\angle AOB = ?$ (2) 扇形 $OAED$ 面積 = ?

$$(1) \angle AOB = \frac{360^\circ}{5} = 72^\circ$$

$$(2) \angle AOD = 72^\circ \times 2 = 144^\circ$$

$$10 \times 10 \times \pi \times \frac{144}{360} = 40\pi$$



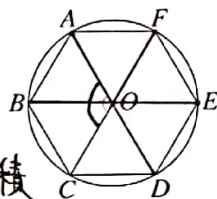
17. 如圖，正六邊形 $ABCDEF$ 的頂點皆在圓 O 上，且圓半徑 = 6，則：(1) $\angle AOC = ?$

(2) $\widehat{AC}$ 的長度 = ? (3) 四邊形 $ABCO$ 的面積

$$(1) \angle AOC = \frac{360^\circ}{6} \times 2 = 120^\circ$$

$$(2) 6 \times 2 \times \pi \times \frac{120}{360} = 4\pi$$

$$(3) \text{ ABCD 面積 } = 2\Delta AOB = 2 \times \frac{\sqrt{3}}{4} \times 6^2 = 18\sqrt{3}$$



18. 如圖，圓 O 為 ΔABC 的外接圓，已知半徑為 6，

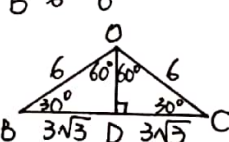
且 $\widehat{AB} : \widehat{BC} : \widehat{AC} = 3 : 4 : 5$ ，求 ΔABC 的面積

$$\angle AOB = 360^\circ \times \frac{3}{12} = 90^\circ$$

$$\angle BOC = 360^\circ \times \frac{4}{12} = 120^\circ$$

$$\angle COA = 360^\circ \times \frac{5}{12} = 150^\circ$$

$$\Delta AOB = \frac{1}{2} \times 6 \times 6 = 18$$

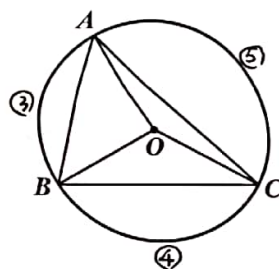


$$\widehat{OD} = \frac{6}{2} = 3$$

$$\widehat{BD} = 3 \times \sqrt{3} = 3\sqrt{3}$$

$$\widehat{BC} = 2 \times 3\sqrt{3} = 6\sqrt{3}$$

$$\Delta BOC = \frac{1}{2} \times 6\sqrt{3} \times 3 = 9\sqrt{3}$$



$$\widehat{AH} = \frac{6}{2} = 3$$

$$\Delta AOC = \frac{1}{2} \times 6 \times 3 = 9$$

$$\Rightarrow 18 + 9\sqrt{3} + 9 = 27 + 9\sqrt{3}$$

19. 已知圓 O_1 的半徑為 18 公分，圓 O_2 的半徑為 12 公分，若圓 O_1 的 $\widehat{AB}$ 長等於圓 O_2 的 $\widehat{CD}$ 長，且 $\widehat{CD} = 72^\circ$ ，求 $\widehat{AB}$ 的度數

設 $\widehat{AB} = x^\circ$

$$\therefore \widehat{AB} \text{ 長} = \widehat{CD} \text{ 長}$$

$$18 \times 2 \times \pi \times \frac{x}{360} = 12 \times 2 \times \pi \times \frac{72}{360}$$

$$3x = 144$$

$$x = 48 \Rightarrow \widehat{AB} = 48^\circ$$