

國八上 週末作業 4-2 解一元二次方程式(公式解) 參考答案 (106.01.02)

1. $x = 3 \pm \sqrt{17}$

2. $x = \frac{7}{2}$ or -3

3. $x = \frac{-5 \pm \sqrt{345}}{20}$

4. $m = 21$

解： $(x+5)^2 = 4$

$$x^2 + 10x + 25 = 4$$

$$x^2 + 10x + 21 = 0 \rightarrow m = 21$$

5. $p = -\frac{3}{2}$ $q = -\frac{19}{4}$

解： $3(x+p)^2 + q = 3(x^2 + 2px + p^2) + q$
 $= 3x^2 + 6px + 3p^2 + q$

$$6p = -9 \rightarrow p = -\frac{3}{2}$$

$$3p^2 + q = 2 \rightarrow 3 \times \left(-\frac{3}{2}\right)^2 + q = 2$$

$$\rightarrow \frac{27}{4} + q = 2 \rightarrow q = -\frac{19}{4}$$

6. $x = -2$ or -6

7. $x = \frac{9 \pm \sqrt{41}}{4}$

8. 無解

9. $x = -\frac{1}{4}$ (重根)

10. $x = \frac{1}{4}$ (重根)

11. $x = \frac{2}{3}$ or $-\frac{3}{4}$

12. $x = 3 \pm \sqrt{17}$

13. $x = \frac{1 \pm \sqrt{241}}{4}$

14. $x = \frac{11 \pm \sqrt{97}}{6}$

15. $x = \frac{1 \pm \sqrt{19}}{9}$

16. $x = 2$ or $\frac{3}{4}$

17. $x = \frac{-1 \pm \sqrt{5}}{2}$

18. $m = 1$

解： $x = 2 \pm \sqrt{3} \rightarrow x - 2 = \pm \sqrt{3}$

$$\rightarrow (x-2)^2 = (\pm\sqrt{3})^2$$

$$\rightarrow x^2 - 4x + 4 = 3$$

$$\rightarrow x^2 - 4x + 1 = 0 \rightarrow m = 1$$

19. $m = -3$

解： $x + \frac{2}{3} = \pm \frac{\sqrt{13}}{3} \rightarrow 3x + 2 = \pm \sqrt{13}$

$$\rightarrow (3x+2)^2 = (\pm\sqrt{13})^2$$

$$\rightarrow 9x^2 + 12x + 4 = 13$$

$$\rightarrow 9x^2 + 12x - 9 = 0$$

$$\rightarrow 3x^2 + 4x - 3 = 0 \rightarrow m = -3$$

20. (1) $b = 10$ $c = 18$

(2) 另一解 $x = -5 + \sqrt{7}$

解： (1) $x = -5 - \sqrt{7} \rightarrow x + 5 = -\sqrt{7}$

$$\rightarrow (x+5)^2 = (-\sqrt{7})^2$$

$$\rightarrow x^2 + 10x + 25 = 7$$

$$\rightarrow x^2 + 10x + 18 = 0$$

$$\rightarrow b = 10 \quad c = 18$$

(2) $x^2 + 10x + 18 = 0$

$$x = \frac{-10 \pm \sqrt{28}}{2} = \frac{-10 \pm 2\sqrt{7}}{2} = -5 \pm \sqrt{7}$$

→ 另一解 $x = -5 + \sqrt{7}$

21. $a = -\frac{18}{5}$

解： $x = \frac{-3 - \sqrt{27}}{5} \rightarrow 5x = -3 - \sqrt{27}$

→ $5x + 3 = -\sqrt{27}$

→ $(5x + 3) = (-\sqrt{27})^2$

→ $25x^2 + 30x + 9 = 27$

→ $25x^2 + 30x - 18 = 0$

→ $5x^2 + 6x - \frac{18}{5} = 0 \rightarrow a = -\frac{18}{5}$

22. $4x^2 + 12x - 11 = 0$

解： $x = \frac{-3 \pm 2\sqrt{5}}{2} \rightarrow 2x = -3 \pm 2\sqrt{5}$

→ $2x + 3 = \pm 2\sqrt{5}$

→ $(2x + 3)^2 = (\pm 2\sqrt{5})^2$

→ $4x^2 + 12x + 9 = 20$

→ $4x^2 + 12x - 11 = 0$

23. (1) $a = 3 - \sqrt{10}$

(2) $-\sqrt{10} + 10$

解：(1) $x = a$ 代入 $a^2 - 6a - 1 = 0$

$$a = \frac{6 \pm \sqrt{40}}{2} = \frac{6 \pm 2\sqrt{10}}{2} = 3 \pm \sqrt{10}$$

∵ $a < 0$ ∴ $a = 3 - \sqrt{10}$

(2) $(3 - \sqrt{10})^2 - 5(3 - \sqrt{10}) + 6$
 $= 9 - 6\sqrt{10} + 10 - 15 + 5\sqrt{10} + 6$
 $= -\sqrt{10} + 10$

24. $m = \pm 8$

解： ∵ 有重根 ($D = 0$)

$$\therefore m^2 - 4 \times (-2) \times (-8) = 0$$

$$m^2 - 64 = 0$$

$$m^2 = 64 \rightarrow m = \pm 8$$

25. $b = -\frac{169}{20}$

解： ∵ 有重根 ($D = 0$)

$$\therefore 13^2 - 4 \times b \times (-5) = 0$$

$$169 + 20b = 0$$

$$20b = -169 \rightarrow b = -\frac{169}{20}$$

26. a 的最大值 = 2

解： ∵ 有兩相異的解 ($D > 0$)

$$\therefore 3^2 - 4 \times 1 \times a > 0$$

$$9 - 4a > 0$$

$$a < \frac{9}{4} = 2\frac{1}{4} \rightarrow a_{\max} = 2$$

27. m 的最小值 = 17

解： ∵ 沒有解 ($D < 0$)

$$\therefore (-7)^2 - 4 \times \frac{1}{2} \times \frac{3}{2} m < 0$$

$$49 - 3m < 0$$

$$m > \frac{49}{3} = 16\frac{1}{3} \rightarrow m_{\min} = 17$$

28. k 的最小值 = 2

解： $2kx^2 - 6x - x^2 + 5 = 0$

$$(2k - 1)x^2 - 6x + 5 = 0$$

∵ 沒有解 ($D < 0$)

$$\therefore (-6)^2 - 4 \times (2k - 1) \times 5 < 0$$

$$36 - 40k + 20 < 0$$

$$40k > 56$$

$$k > \frac{7}{5} = 1\frac{2}{5} \rightarrow k_{\min} = 2$$